Neutrino Mixing and Oscillations in 1999 and Beyond [a]

S. T. Petcov [b]

Scuola Internazionale Superiore di Studi Avanzati, I-34014 Trieste, Italy, and
Istituto Nazionale di Fisica Nucleare, Sezione di Trieste, I-34014 Trieste, Italy

The interpretation of the existing experimental eveidences for oscillations of neutrinos in schemes with three and four neutrino mixing is reviewed. Forms of the lepton mixing matrix allowed by the neutrino oscillation data are considered. The possible neutrino mass spectra compatible with the observations are analyzed. The possibility to obtain information about the neutrino mass spectrum from the future ^{3}H $\beta-$decay and $(\beta\beta)_{0\nu}-$decay experiments is also considered.

1 Introduction

At present there exist strong evidences that the flavour neutrinos, ν_e and ν_μ take part in neutrino oscillations[1] (see also[2,3]) or undergo transitions in matter[4] into neutrinos of different type. They come primarily from the experiments with solar[5,6] and atmospheric neutrinos[6,7]. Indications for neutrino oscillations have been obtained also in the LSND neutrino oscillation experiment[8]: the anomalous events observed in this experiment can be interpreted as being due to small mixing $\bar{\nu}_\mu \leftrightarrow \bar{\nu}_e$ oscillations.

These evidences suggest that neutrinos have nonzero masses and that lepton mixing is present in the weak charged lepton current. If definitely established, the existence of nonzero neutrino masses and of lepton mixing will have profound implications for our understanding of the elementary particle interactions. It entails a number of important and not easy to answer questions (see, e.g., [9]) as well. Here we give a possible short list. i) The number of massive neutrinos can be equal to, or be greater than, the number of flavour neutrinos. Which of the two possibilities is realized, in other words, are there sterile neutrinos, ν_s? What is the number of neutrinos with definite mass? What is it determined by? ii) The neutrino oscillation data does not permit to

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determine the absolute values of the neutrino masses. What are they, i.e., what is the neutrino mass spectrum? iii) What are the values of the elements of of the lepton mixing matrix? Does the lepton mixing matrix contain nontrivial CP-violation phases? Are there CP- violation effects in neutrino oscillations? iv) Neutrinos with definite mass can be Dirac or Majorana particles. Which of the two possibilities is realized and why? v) Massive neutrinos and lepton mixing imply that the additive lepton charges - the electron L_e, the muon L_μ and the tauon L_τ, are not conserved by the elementary particle interactions. Do lepton number non-conserving processes other than neutrino oscillations, as like $\mu^- \to e^- + \gamma$, $\mu^- \to e^- + e^+ + e^-$, etc. exist at observable level? This list can be continued. It is clear that by proving the existence of nonzero neutrino mass and of lepton mixing one would establish the existence of a whole "new world" in elementary particle physics. There are strong evidences that this "new world" indeed exists and we are just beginning to explore it.

In the present article we shall review the evidences for oscillations of neutrinos. The interpretation of the solar and atmospheric neutrino data in a scheme with three-neutrino mixing will be considered. We shall discuss briefly the new effect of maximal enhancement of the transitions in the Earth of the solar and atmospheric neutrinos which cross the Earth core on the way to the detectors. Further, results obtained assuming four neutrino mixing and including the LSND indications for oscillations in the analysis of the data will be reviewed. Rather simple patterns of lepton mixing emerge from these analyses. We will consider next the possibility to obtain information about the neutrino mass spectrum from the future ^{3}H $\beta-$decay and $(\beta\beta)_{0\nu}-$decay experiments.

2   The Data or the "Initial" and the" Boundary" Conditions of the Analysis

We have strong indications for vacuum $\nu_e \leftrightarrow \nu_{\mu,\tau}$ oscillations (VO), or MSW $\nu_e \to \nu_{\mu,\tau}$ or $\nu_e \to \nu_s$ transitions, of the solar neutrinos from the mean event rate solar neutrino data (see, e.g., [10,11]). If one uses the data as published by the Homestake, SAGE, GALLEX and Super-Kamiokande collaborations in the analyses of the VO or MSW hypotheses, one finds that i) the $\nu_e \leftrightarrow \nu_s$ oscillations in vacuum, ii) the $\nu_e \to \nu_s$ large mixing angle (LMA) MSW transitions of solar neutrinos, as well as iii) a universal energy-independent suppression of the solar neutrino flux, are disfavored (in some of the cases - strongly) by the data [12,13,14,15,10,11]. This conclusion is based, to large extent, on the result of the Homestake experiment when compared with the results of the other solar neutrino experiments. If, e.g., the systematic error in the ^{37}Ar production rate, reported by the Homestake collaboration, is arbitrarily

increased by a factor of ~ 3, the above results change and, for instance, the hypothesis of a constant suppression of the solar neutrino flux by a factor of ~ 0.5 becomes no longer strongly disfavored by the data. However, we do not see at present any reasons for changing the value of the systematic uncertainty in the Homestake data, given by the Homestake collaboration, and will not consider further the indicated three disfavored possibilities. We will not discuss also the possible solution of the solar neutrino problem based on the hypothesis of existence of new neutrino flavour changing and neutrino flavour conserving but flavour non-symmetric neutral current interactions [16], or of violation of the weak equivalence principle [17], etc. Both have difficulties in explaining the atmospheric neutrino data [18]. We will be interested in the simplest solutions of the solar neutrino problem, which can be incorporated naturally in schemes providing explanation of the atmospheric neutrino anomalies as well. Following these guiding rules we are left with just four generic possibilities: the large mixing vacuum oscillations $\nu_e \leftrightarrow \nu_{\mu,\tau}$, the small and the large mixing MSW $\nu_e \rightarrow \nu_{\mu,\tau}$ transitions and the small mixing (SMA) MSW $\nu_e \rightarrow \nu_s$ transitions.

According to the recent analyses [10,11], the two-neutrino $\nu_e \leftrightarrow \nu_{\mu,\tau}$ oscillations in vacuum provide a description (at 95% C.L.) of the solar neutrino data for values of the two vacuum oscillation parameters, Δm^2 and $\sin^2 2\theta$, belonging approximately to the region:

$$5.0 \times 10^{-11} \text{eV}^2 \lesssim \Delta m^2 \lesssim 5.0 \times 10^{-10} \text{eV}^2, \tag{1a}$$

$$0.65 \lesssim \sin^2 2\theta \leq 1.0. \tag{1b}$$

The SMA and LMA MSW $\nu_e \rightarrow \nu_{\mu,\tau}$ transition solutions take place for values of Δm^2 and $\sin^2 2\theta$ from the intervals

$$4.0 \times 10^{-6} \text{ eV}^2 \lesssim \Delta m^2 \lesssim 9.0 \times 10^{-6} \text{ eV}^2, \tag{2a}$$

$$10^{-3} \lesssim \sin^2 2\theta \leq 10^{-2}. \tag{2b}$$

and

$$2.0 \times 10^{-5} \text{ eV}^2 \lesssim \Delta m^2 \lesssim 2.0 \times 10^{-4} \text{ eV}^2, \tag{3a}$$

$$0.65 \lesssim \sin^2 2\theta \leq 1.0. \tag{3b}$$

while the SMA MSW $\nu_e \rightarrow \nu_s$ transition solution is realized for

$$3.0 \times 10^{-6} \text{ eV}^2 \lesssim \Delta m^2 \lesssim 8.0 \times 10^{-6} \text{ eV}^2, \tag{4a}$$

$$1.5 \times 10^{-3} \lesssim \sin^2 2\theta \lesssim 1.2 \times 10^{-2}. \tag{4b}$$

Although these results are obtained utilizing the standard solar model predictions of ref. [19] for the fluxes of the pp, pep, ^{7}Be, ^{8}B and CNO neutrinos, they

are rather stable with respect to variation of the fluxes within their estimated uncertainty ranges (see, e.g.,[11,14,20]) and so is the existence of the four generic solutions. Moreover, the relative magnitudes of $\sin^2 2\theta$ and of Δm^2 for the four solutions remain unchanged with respect to such variations: we always have, e.g., $\Delta m^2_{VO} \ll \Delta m^2_{SMA} < \Delta m^2_{LMA}$ and $\sin^2 2\theta_{SMA} \ll \sin^2 2\theta_{LMA} \sim \sin^2 2\theta_{VO}$, where Δm^2_i and $\sin^2 2\theta_i$, i=VO,SMA,LMA, are the values of the two parameters required by the VO and the SMA and LMA MSW solutions.

In spite of the strong indications from the mean event rate data that the solar neutrinos take part in one of the four types of oscillations/transitions discussed above, none of the physical effects considered to be the "hallmarks" of the four solutions, have been observed so far. More concretely, i) the specific seasonal variation effect predicted in the case of the VO solution (see the second and the third articles quoted in [3] and, e.g.,[13,21,22] and the references quoted therein) , ii) the day-night (D-N) effect which should take place in the case of the MSW solutions (see, e.g.,[10,11,23,24] and the references quoted therein) and iii) the characteristic distortions of the $e^- -$spectrum measured by the Super-Kamiokande (SK) collaboration, which are predicted in the cases of the VO and SMA MSW solutions (see, e.g.,[13,10,11]), have not been observed [6]. The unexpected rise of the spectrum at $E_e \gtrsim 12.5$ MeV seen in the SK experiment, E_e being the recoil $e-$ total energy, may indicate an anomalously large contribution in the above energy range from the so-called "hep" solar neutrinos [25] produced in the reaction $p +^3 He \to^4 He + e^+ + \nu_e$. Below 12.5 MeV the measured spectrum is compatible with the absence of distortions. The whole SK data on the $e^- -$spectrum, including the points above 12.5 MeV, seem to favor the VO solution with [21,6] "large" $\Delta m^2 \cong 4.3 \times 10^{-10} \text{eV}^2$ and $\sin^2 2\theta \cong 0.9$, and the LMA MSW solution. Obviously, more precise data is needed to resolve these ambiguities and/or to rule out some of the four solutions.

Very strong evidences for oscillations of the atmospheric ν_μ and $\bar{\nu}_\mu$ neutrinos [c] have been obtained, as is well-known, in the SK experiment [6]. These include the measured nonzero Up - Down asymmetry (a ~ 7 s.d. effect!) and the observed substantial Zenith angle dependence of the rates of the sub-GeV and multi-GeV $\mu-$like events. No similar effects were observed in the samples of $e-$like events. This implies that if the atmospheric ν_μ take part in oscillations, which is the only plausible explanation of the SK $\mu-$like data available at present, the dominant oscillation modes should not include the ν_e. We are therefore left with two possibilities: the dominant oscillations can be either of the type $\nu_\mu \leftrightarrow \nu_\tau$ or $\nu_\mu \leftrightarrow \nu_s$. The values of the two-neutrino oscillations

[c]We will often use the name "atmospheric ν_μ (or ν_e)" to indicate both neutrinos and antineutrinos.

parameters following (at 90% C.L.) from the data are the following[6]:

$$\nu_\mu \leftrightarrow \nu_\tau : \quad 10^{-3} \text{ eV}^2 \lesssim \Delta m^2 \lesssim 8.0 \times 10^{-2} \text{ eV}^2, \quad (5a)$$

$$0.86 \lesssim \sin^2 2\theta \leq 1.0, \quad (5b)$$

$$\nu_\mu \leftrightarrow \nu_s : \quad 2 \times 10^{-3} \text{ eV}^2 \lesssim \Delta m^2 \lesssim 7.0 \times 10^{-2} \text{ eV}^2, \quad (6a)$$

$$0.86 \lesssim \sin^2 2\theta \leq 1.0. \quad (6b)$$

The L/E or the L (or Zenith angle) independent suppression of the atmospheric ν_μ flux, where E is the neutrino energy and L is the length of the path the neutrinos travel before reaching the detector, are incompatible with the atmospheric neutrino data[18]. In particular, the hypothesis of the atmospheric neutrino decay, or of gravitationally induced oscillations[17] of the atmospheric ν_μ, are disfavored by the SK atmospheric neutrino data (including the data on the through-going and stopping muons).

Indications for neutrino oscillations have been obtained also in the LSND experiment[8]: the anomalous events observed by the LSND collaboration can be interpreted as being due to $\bar{\nu}_\mu \leftrightarrow \bar{\nu}_e$ oscillations with Δm^2 lying in the interval $\sim (0.3 \text{ - } 10) \text{ eV}^2$ and $\sin^2 2\theta \sim \text{few} \times 10^{-3}$. The KARMEN collaboration, which performs a search for the indicated type of oscillations in the same energy range, but at an approximately two times smaller distance between the neutrino source and the detector than in the LSND experiment, has not observed anomalous events in excess of their estimated background[26]. The KARMEN results cannot completely rule out, however, the possibility that the LSND anomalous events are due to small mixing $\bar{\nu}_\mu \leftrightarrow \bar{\nu}_e$ oscillations with values of Δm^2 in the indicated range, although they exclude a large fraction of the region in the $\Delta m^2 - \sin^2 2\theta$ plane suggested by the LSND data.

An important neutrino oscillation constraint has been obtained in the CHOOZ experiment[27] with reactor $\bar{\nu}_e$. The CHOOZ collaboration has not observed a disappearance of the $\bar{\nu}_e$ at a distance of ~ 1 km from the reactor. Interpreted in terms of two-neutrino oscillations this result implies, e.g., that

$$for \ \Delta m^2 \geq 1.5 \times 10^{-3} \text{ eV}^2, \quad \sin^2 2\theta < 0.22 \ (90\% \ C.L.). \quad (7)$$

Rather stringent constraints on the neutrino masses have been derived in the ^{3}H $\beta-$decay experiments as well as in the experiments searching for neutrinoless double beta $((\beta\beta)_{0\nu}-)$ decay, $(A, Z) \to (A, Z+2) + e^- + e^-$. The latter is allowed if the neutrinos with definite mass in vacuum are Majorana particles. The upper limit on the electron neutrino mass, $m(\nu_e)$, obtained in the Moscow[28] and Mainz[29] ^{3}H $\beta-$decay experiments reads:

$$m(\nu_e) < 2.5 \text{ eV}, \quad m(\nu_e) < 2.9 \text{ eV} \quad (95\% \ C.L.). \quad (8)$$

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There are plans to improve these limits by a factor of ~ 3, and thus to probe the 1 eV region [28,29]. The best limit on the effective neutrino mass parameter $< m_\nu >$ extracted form the data on the $(\beta\beta)_{0\nu}-$decay (see further), has been derived in the Heidelberg - Moscow ^{76}Ge experiment [30]:

$$| < m_\nu > | \ < (0.5 - 1.0) \text{ eV} \quad (90\% \ C.L.). \tag{9}$$

The range in eq. (9) reflects the uncertainty in the calculations of the corresponding nuclear matrix elements. This limit is planned to be improved at least by a factor of $\sim (3 - 4)$ in the ongoing Heidelberg - Moscow experiment and in the NEMO experiment which is under preparation. Two experiments have been proposed, GENIUS and CUORE [31], with a projected sensitivity to values of $| < m_\nu > |$ as small as $\sim (5 \times 10^{-2} - 10^{-3})$ eV.

Recent developments in the field of astrophysics and cosmology suggesting the existence of a nonzero cosmological constant imply, in particular, that the neutrinos with masses whose sum is $\sim (4 - 6)$ eV are no longer required to provide the hot dark matter component in the Universe [32]. Nevertheless, neutrinos having a mass exceeding ~ 1 eV can be cosmologically relevant.

To summarize, the solution of the solar neutrino problem, the explanation of the atmospheric neutrino data and of the LSND results suggest three very different values of the parameter Δm^2, namely, $\Delta m^2_\odot \lesssim 10^{-4}$ eV^2, $\Delta m^2_{atm} \cong (10^{-3} - 10^{-2})$ eV^2, $\Delta m^2_{LSND} \cong (0.3 - 10.0)$ eV^2. Correspondingly, we have: $\Delta m^2_\odot \ll \Delta m^2_{atm} < (\ll)\Delta m^2_{LSND}$. The minimal scheme having three independent values of Δm^2 is, obviously, a scheme with four mixed neutrinos. The LEP data on the number of the light neutrinos coupled to the Z^0-boson and the cosmological constraints on the number of light neutrinos imply that the needed fourth weak-eigenstate neutrino must be a sterile neutrino, ν_s.

In Section 3 we will describe the solar and atmospheric neutrino oscillations/transitions in schemes with three-neutrino mixing, while in Section 4 we will consider examples of schemes with four-neutrino mixing which can accommodate also the $\bar{\nu}_\mu \leftrightarrow \bar{\nu}_e$ oscillations suggested by the LSND data.

3 Three-Flavour Neutrino Mixing

Consider the case of three-flavour neutrino mixing,

$$|\nu_l > \ = \sum_{k=1}^{3} U^*_{lk}|\nu_k >, \qquad l = e, \mu, \tau, \tag{10}$$

where $|\nu_l >$ is the state vector of the (left-handed) flavour neutrino ν_l (with momentum $\overrightarrow{p}$), $|\nu_k >$ is the state vector of a neutrino ν_k possessing a definite

mass m_k (and momentum $\overrightarrow{p}$), $m_k \neq m_j$, $k \neq j = 1, 2, 3$, $m_1 < m_2 < m_3$, and U is a 3×3 unitary matrix – the lepton mixing matrix. It is natural to assume in this case that one of two independent neutrino mass-squared differences, say, Δm_{21}^2, is relevant for the VO or MSW solutions of the solar neutrino problem, and has a value in one of the intervals given in eqs. (1a) - (4a), while Δm_{31}^2 is responsible for the dominant oscillations of the atmospheric ν_μ, $\nu_\mu \leftrightarrow \nu_\tau$. and lies in the interval (5a). For the indicated values of Δm_{21}^2 and Δm_{31}^2 and

$$\Delta m_{21}^2 = \Delta m_\odot^2 \ll \Delta m_{atm}^2 = \Delta m_{31}^2 , \tag{11}$$

the relevant three-neutrino VO or MSW transition probabilities, describing the solar neutrino conversion into another flavour neutrino, as well as the three-neutrino oscillation probabilities for the atmospheric neutrinos, reduce effectively to two-neutrino oscillation/transition probabilities[33,34,35]. We have:

$$P_\odot(\nu_e \to \nu_e) \cong |U_{e3}|^4 + (1 - |U_{e3}|^2)^2 \ P_\odot^{2\nu}(\nu_e \to \nu_e), \tag{12}$$

$$P_{atm}(\nu_\mu \to \nu_\tau) = P_{atm}(\bar{\nu}_\mu \to \bar{\nu}_\tau) \cong 2|U_{\mu 3}|^2 |U_{\tau 3}|^2 \ \left(1 - \cos\frac{\Delta m_{31}^2}{2E}L\right), \tag{13}$$

$$P_{atm}^{vac}(\nu_\mu(\nu_e) \to \nu_e(\nu_\mu)) \cong 2|U_{\mu 3}|^2 |U_{e3}|^2 \ \left(1 - \cos\frac{\Delta m_{31}^2}{2E}L\right), \tag{14}$$

$$P_{atm}^{vac}(\nu_\mu(\nu_e) \to \nu_e(\nu_\mu)) \cong P_{atm}^{vac}(\bar{\nu}_\mu(\bar{\nu}_e) \to \bar{\nu}_e(\bar{\nu}_\mu)),$$

$$P_{CHOOZ}(\bar{\nu}_e \to \bar{\nu}_e) \cong 1 - 2|U_{e3}|^2(1 - |U_{e3}|^2) \ \left(1 - \cos\frac{\Delta m_{31}^2}{2E}L\right). \tag{15}$$

Here $P_\odot(\nu_e \to \nu_e)$ is the solar ν_e survival probability if (10) and (11) hold, $P_\odot^{2\nu}(\nu_e \to \nu_e) \equiv P_\odot^{2\nu}(\Delta m_{21}^2/2E, \sin^2 2\theta_{12}, |U_{e3}|^2)$ is the VO or MSW two-neutrino mixing solar ν_e survival probability, where

$$\sin^2 2\theta_{12} = 4\frac{|U_{e1}|^2 \ |U_{e2}|^2}{(|U_{e1}|^2 + |U_{e2}|^2)^2}, \quad \cos 2\theta_{12} = \frac{|U_{e1}|^2 - |U_{e2}|^2}{|U_{e1}|^2 + |U_{e2}|^2}, \tag{16}$$

and the other notations are self-explanatory. In the case of the VO solution the probability $P_\odot^{2\nu}(\nu_e \to \nu_e)$ does not depend on $|U_{e3}|^2$ and is given by the standard two-neutrino mixing expression with Δm_{21}^2 and $\sin^2 2\theta_{12}$ playing the role of the two oscillation parameters. If the solar ν_e take part in MSW transitions, the dependence of $P_\odot^{2\nu}(\nu_e \to \nu_e)$ on $|U_{e3}|^2$ amounts to the change of the matter term[34,35]

$$\sqrt{2}G_F N_e \to \sqrt{2}G_F N_e(1 - |U_{e3}|^2), \tag{17}$$

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N_e being the electron number density, in the standard expression for the two-neutrino mixing survival probability. Let us note finally that the expression for the probability $P_{atm}^{vac}(\nu_e \to \nu_\tau) = P_{atm}^{vac}(\bar{\nu}_e \to \bar{\nu}_\tau)$ can be obtained from the expression in the right-hand side of eq. (13) by replacing $|U_{\mu 3}|^2$ with $|U_{\tau 3}|^2$.

Under the condition (11), the solar neutrino survival probability (12) depends only on the elements of the first row of the lepton mixing matrix, i.e., on $|U_{ei}|^2$, i=1,2,3, while the oscillations of the atmospheric ν_μ and ν_e are controlled by the elements of the third column of U, $|U_{l3}|^2$, $l = e, \mu, \tau$. The other elements of U are not accessible to direct experimental determination.

The CHOOZ limit (7) and analysis of the solar neutrino data based on eq. (12) imply that $|U_{e3}|^2$ has to be small: for $\Delta m_{31}^2 \gtrsim 1.5 \times 10^{-3}$ eV^2 one has

$$|U_{e3}|^2 \lesssim 0.05. \tag{18}$$

This may be indicating that $|U_{e3}| \ll 1$, or even that $|U_{e3}| \cong 0$. The lepton mixing matrix takes the particularly simple form of *bi-maximal mixing* (see, e.g., [37,38] and the references quoted therein) in the case of the VO solution of the solar neutrino problem if $|U_{e3}| \cong 0$ and we assume that $\sin^2 2\theta_{12} = 1$ and $|U_{\mu 3}|^2 = |U_{\tau 3}|^2$:

$$U \cong \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \\ \frac{1}{2} & -\frac{1}{2} & \frac{1}{\sqrt{2}} \end{pmatrix}. \tag{19}$$

In this case there will be no CP-violation in the oscillations of neutrinos in vacuum; and if the neutrinos with definite mass ν_i, $i = 1, 2, 3$, are Dirac particles there will be no CP-violation at all in the lepton sector. If, however, ν_i are massive Majorana neutrinos, there can be CP-violation related effects in processes which are associated with the Majorana nature of the ν_i's, as like the $(\beta\beta)_{0\nu}$−decay [9,36], etc. The solar ν_e will oscillate with equal probabilities into ν_μ and ν_τ and the atmospheric ν_e will not oscillate over the distances probed by the atmospheric neutrino experiments ($L \lesssim 12800$ km).

The above conclusions will be approximately valid if $|U_{e3}| \neq 0$, but $|U_{e3}| \ll 1$. In particular, the CP-violation effects in neutrino oscillations will be strongly suppressed. Further, one can determine the elements of U from the analysis of the solar and atmospheric neutrino data. There are three possible solutions of the solar neutrino problem - the MSW $\nu_e \to \nu_s$ transition solution cannot be realized in the case of (10). Utilizing a standard parametrization of U we have:

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & U_{e3} \\ -s_{12}c_{23} - c_{12}s_{23}U_{e3}^* & c_{12}c_{23} - s_{12}s_{23}U_{e3}^* & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}U_{e3}^* & -c_{12}s_{23} - s_{12}c_{23}U_{e3}^* & c_{23}c_{13} \end{pmatrix}$$

$$\cong \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & << 1 \\ -s_{12}c_{23} & c_{12}c_{23} & s_{23} \\ s_{12}s_{23} & -c_{12}s_{23} & c_{23} \end{pmatrix}, \tag{20}$$

where $c_{ij} \equiv \cos\theta_{ij}$, $s_{ij} \equiv \sin\theta_{ij}$ and $U_{e3} = s_{13}e^{-i\delta_{13}}$, δ_{13} being the Dirac CP-violation phase and we have not written explicitly the possible Majorana CP-violation phases (see, e.g., [9]). The angles θ_{12} and θ_{23} in (20) are fixed (with a known ambiguity) within rather narrow ranges by the solar and the atmospheric neutrino data and this determines all the elements of the lepton mixing matrix in eq. (20). One finds for the three solutions of the solar neutrino problem:

$$VO: \quad |s_{12}| \cong 0.48 - 0.71, \tag{21a}$$

$$SMA\ MSW: \quad s_{12} \cong 0.02 - 0.05, \tag{21b}$$

$$LMA\ MSW: \quad s_{12} \cong 0.30 - 0.55. \tag{21c}$$

The atmospheric neutrino data implies:

$$\nu_\mu \leftrightarrow \nu_\tau: \quad |s_{23}| \cong 0.50 - 0.71. \tag{22}$$

Clearly, the lepton mixing matrices corresponding to eqs. (21a) - (22) are very different from the quark mixing matrix.

It follows from the above discussion that under the condition (11), the magnitude of $|U_{e3}|$ controls, in particular, the $\nu_e \leftrightarrow \nu_{\mu,\tau}$ and $\nu_\mu \leftrightarrow \nu_e$ oscillations of the atmospheric and terrestrial (i.e., "man made") neutrinos as well as the magnitude of the CP-violation effects in the oscillations of neutrinos and in the lepton sector, in general. It would be extremely important to obtain better experimental limits on, or determine the value of $|U_{e3}|$. The long base-line neutrino oscillations experiments MINOS and ICARUS[7] are envisaged to be sensitive to values of $|U_{e3}|^2$ as small as $\sim 5 \times 10^{-3}$.

There is an additional very interesting new physical effect related to the $|U_{e3}|$. We have notice that the latter "drives" the sub-dominant $\nu_e \leftrightarrow \nu_{\mu,\tau}$ and $\nu_\mu \leftrightarrow \nu_e$ oscillations of the atmospheric ν_μ and ν_e. As was pointed out in [39], for the neutrinos passing through the Earth, these oscillations can be strongly amplified by a new resonance-like mechanism, which differs from the MSW one and takes place when the neutrinos cross the Earth core. The new mechanism can cause a total neutrino conversion [40]. At small mixing angles ($\sin^2 2\theta \lesssim 0.10$), the maxima due to this new enhancing mechanism in $P(\nu_\mu \rightarrow \nu_e)$ and $P(\nu_e \rightarrow \nu_{\mu(\tau)})$ are absolute maxima and dominate in these probabilities: they are considerably larger than the local maxima of $P(\nu_\mu \rightarrow \nu_e)$ and $P(\nu_e \rightarrow \nu_{\mu(\tau)})$, associated with the MSW effect taking place

in the Earth core (mantle) [d]. The mixing angle which is relevant for the magnitude of the enhancement, is determined in the case of interest by $|U_{e3}|^2$ (see further). Even at small mixing angles the enhancement is relatively wide [39,40,41,42] in the Nadir (or Zenith) angle [e], h, and in the neutrino energy E - it is somewhat wider than the MSW resonance, and therefore can produce observable effects. As was shown in [40], the new mechanism of enhancement of the neutrino transitions in the Earth is of interference nature: it is caused by a maximal constructive interference between the probability amplitudes of the neutrino transitions in the Earth mantle and in the Earth core. Thus, the effect has nothing to do with the parametric resonance in the neutrino transitions, discussed in [43] and possible in a medium with periodic change of density.

The conditions for a total neutrino conversion due to the new effect, $P(\nu_\mu \to \nu_e) = 1$, in the two-neutrino mixing case include specific relations between the phase differences the neutrino energy-eigenstates acquire after crossing the Earth mantle, $2\phi'$, and the core, $2\phi''$ and the mixing angles in matter in the mantle, θ'_m, and in the core, θ''_m. For the two-neutrino $\nu_{\mu(e)} \to \nu_{e(\mu;\tau)}$, $\nu_e \to \nu_s$ and $\bar{\nu}_\mu \to \bar{\nu}_s$ transitions ($\Delta m^2 \cos 2\theta > 0$) they read [40]:

$$\tan \phi' = \pm \sqrt{\frac{-\cos 2\theta''_m}{\cos(2\theta''_m - 4\theta'_m)}}, \quad \tan \phi'' = \pm \frac{\cos 2\theta'_m}{\sqrt{-\cos(2\theta''_m)\cos(2\theta''_m - 4\theta'_m)}},$$
(23)

where the signs are correlated [f]. It is quite remarkable that these conditions are satisfied [40,39] for the Earth-core-crossing solar and atmospheric neutrinos.

The $\nu_e \to \nu_{\mu;\tau}$ and $\nu_\mu \to \nu_e$ transition probabilities in the Earth are given under the conditions (10) - (11) by (see, e.g., [35]):

$$P_E^{3\nu}(\nu_\mu(\nu_e) \to \nu_e(\nu_\tau)) \cong \frac{|U_{\mu(\tau)3}|^2}{1 - |U_{e3}|^2} \; P_E^{2\nu}(\Delta m_{31}^2, \sin^2 2\theta_{13}),$$

where $P_E^{3\nu}(\nu_e \to \nu_\mu) \cong P_E^{3\nu}(\nu_\mu \to \nu_e)$, $P_E^{2\nu}(\Delta m_{31}^2, \sin^2 2\theta_{13})$ is a known two-neutrino transition probability for the Earth-core-crossing neutrinos [g] and $\sin^2 2\theta_{13} \equiv 4|U_{e3}|^2(1 - |U_{e3}|^2)$. For the fluxes of the atmospheric $\nu_{e,\mu}$ with energy E, crossing the Earth along a trajectory with Zenith angle θ_z before

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[d]The effect of the new enhancement is less dramatic at large mixing angles. The enhancement is present and dominates also in the $\nu_2 \to \nu_e$ transitions in the case of $\nu_e - \nu_{\mu(\tau)}$ or $\nu_e - \nu_s$ mixing and in the $\nu_e \to \nu_s$ and $\bar{\nu}_\mu \to \bar{\nu}_s$ transitions at small mixing angles [39,40,41,42].

[e]The Nadir angle determines uniquely the neutrino trajectory through the Earth.

[f]The conditions for the $\nu_2 \to \nu_e$ transitions can formally be obtained from eq. (23) by replacing $2\theta''_m$ and $2\theta'_m$ in the expressions for ϕ' and ϕ'' with $(2\theta''_m - \theta)$ and $(2\theta'_m - \theta)$.

[g]An analytic expression for $P_E^{2\nu}(\Delta m_{31}^2, \sin^2 2\theta_{13})$ can be found in [39].

reaching the detector we get [41,42]:

$$\Phi_{\nu_e} \cong \Phi_{\nu_e}^0 \left(1 + [s_{23}^2 \ r(E,\theta_z) - 1] \ P_E^{2\nu}(\Delta m_{31}^2, \sin^2 2\theta_{13})\right), \tag{24}$$

$$\Phi_{\nu_\mu} \cong \Phi_{\nu_\mu}^0 (1 + s_{23}^4 \ [(s_{23}^2 \ r(E,\theta_z))^{-1} - 1] \ P_E^{2\nu}(\Delta m_{31}^2, \sin^2 2\theta_{13})$$
$$- 2c_{23}^2 s_{23}^2 \ [1 - Re \ (e^{-i\kappa} A_E^{2\nu}(\Delta m_{31}^2, \sin^2 2\theta_{13}))]), \tag{25}$$

where $\Phi_{\nu_{e(\mu)}}^0 = \Phi_{\nu_{e(\mu)}}^0(E,\theta_z)$ is the $\nu_{e(\mu)}$ flux in the absence of oscillations, $s_{23}^2 \equiv |U_{\mu 3}|^2/(1-|U_{e3}|^2)$, $r(E,\theta_z) \equiv \Phi_{\nu_\mu}^0/\Phi_{\nu_e}^0$, $A_E^{2\nu}(\Delta m_{31}^2, \sin^2 2\theta_{13})$ is a known amplitude of two-neutrino transitions in the Earth, and $\kappa \equiv \kappa(\Delta m_{31}^2, \sin^2 2\theta_{13})$ is a known phase factor. Analytic expressions for $A_E^{2\nu}$ and κ are given in [41,42].

The probability $P_E^{2\nu}(\Delta m_{31}^2, \sin^2 2\theta_{13})$ can be strongly enhanced to values ~ 1 by the new resonance-like mechanism. At $\sin^2 2\theta_{13} \lesssim 0.2$ and for values of Δm_{31}^2 suggested by the SK data, the enhancement takes place [39] for the ν_e and ν_μ with energies $\sim (1.0 - 10.0)$ GeV, which contribute either to the sub-GeV or to the multi-GeV $e-$like and $\mu-$like SK event samples. At small mixing angles the enhancement holds practically for all neutrino trajectories through the Earth core. The new effect can produce an excess of e-like events in the region $-1 \leq \cos\theta_z \lesssim -0.8$, θ_z being the Zenith angle, in the multi-GeV (or a smaller one - in the sub-GeV) sample of events. Actually, the effect may have already manifested itself [39,41,42], producing at least part of the strong Zenith angle dependence observed in the multi-GeV $\mu-$like event rate in the SK experiment [h]. The multi-GeV SK data can be used to obtain information on the value of $\sin^2 2\theta_{13}$ and thus of $|U_{e3}|^2$.

Let add finally that the new enhancement mechanism is operative also in the transitions of the Earth-core-crossing solar neutrinos [39,40]. This has important implications for the tests of the MSW $\nu_e \to \nu_{\mu(\tau)}$ transition solutions of the solar neutrino problem, as discussed in detail in [39,24].

4 Scenarios with Four-Neutrino Mixing

In the case of four-neutrino mixing we have

$$|\nu_\alpha> \ = \sum_{k=1}^{4} U_{lk}^* |\nu_k>, \qquad \alpha = e, \mu, \tau, s \tag{26}$$

where U is now a 4×4 unitary matrix. This may seem to be a rather messy case with a large number of possible relations between the three independent

[h]The new effect should also be present in the $\bar{\nu}_\mu \leftrightarrow \bar{\nu}_s$ transitions of the atmospheric multi-GeV $\bar{\nu}_\mu$'s both at small, intermediate and large mixing angles, if the atmospheric neutrinos undergo such transitions.

neutrino mass-squared differences and a large number of mixing parameters: the matrix U contains now 6 mixing angles and 3 Dirac type CP-violation phases. However, as was shown in [44], only two possibilities, in what regards the relations between the different Δm^2, are compatible with the existing data, including the LSND result and the constraints on the neutrino oscillations parameters obtained in the accelerator experiments:

$$\Delta m_{43}^2 = \Delta m_\odot^2 \ll \Delta m_{21}^2 = \Delta m_{atm}^2 \ , \qquad (A)$$

with $\Delta m_{LSND}^2 = \Delta m_{41}^2 \cong \Delta m_{42}^2 \cong \Delta m_{31}^2 \cong \Delta m_{32}^2$, and [45]

$$\Delta m_{21}^2 = \Delta m_\odot^2 \ll \Delta m_{43}^2 = \Delta m_{atm}^2 \ , \qquad (B)$$

with $\Delta m_{LSND}^2 = \Delta m_{41}^2$, etc. The matrix U takes a particularly simple form if one implements the *standard* Big Bang Nucleosynthesis (BBN) constraint on the number of light neutrinos, N_ν: $N_\nu \lesssim 4$. One finds [44] in the case (A):

$$U \cong \begin{pmatrix} 0 & 0 & \cos\beta & \sin\beta \\ \cos\gamma & \sin\gamma & 0 & 0 \\ -\sin\gamma & \cos\gamma & 0 & 0 \\ 0 & 0 & -\sin\beta & \cos\beta \end{pmatrix} \qquad (27)$$

where β and γ are determined by the solar and atmospheric neutrino data, respectively. The small mixing responsible for the LSND effect can be accounted for by a minor modification of the matrix (27). The solar neutrino problem is solved in the scheme (A) by SMA MSW $\nu_e \to \nu_s$ transitions, while the dominant oscillations of the atmospheric ν_μ are of the type $\nu_\mu \leftrightarrow \nu_\tau$: the $\nu_\mu \leftrightarrow \nu_s$ transitions are strongly suppressed. The mixing matrix in the case (B) has a similar structure: it can be obtained from eq. (27) by interchanging the first (second) and the third (fourth) columns. The solutions of the solar and atmospheric neutrino problems are the same.

5 The Neutrino Mass Spectrum

The neutrino oscillation experiments or the interpretation of a given set of data (solar, atmospheric, LSND) in terms of neutrino oscillations or MSW transitions, does not provide information about the absolute values of the neutrino masses. In the scheme with three-neutrino mixing, for example, there are three possible types of neutrino mass spectrum and all of them lead to the same neutrino oscillation phenomenology. We can have a hierarchical spectrum,

$$m_1 \ll m_2 \ll m_3, \qquad (28)$$

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there could be two quasi-degenerate neutrinos,

$$m_1 \cong m_2 \ll m_3, \quad or \quad m_1 \ll m_2 \cong m_3, \tag{29}$$

or three quasi-degenerate neutrinos,

$$m_1 \cong m_2 \cong m_3. \tag{30}$$

In the cases (28) - (29) one has to identify Δm_{31} with Δm_{atm}^2 in order to explain the atmospheric neutrino data. Correspondingly, we have $m_3 \cong \sqrt{\Delta m_{atm}^2}$ and all neutrino masses cannot exceed the value $\sqrt{\Delta m_{atm}^2}$: $m_i \lesssim (0.03 - 0.10)$ eV. The values of the neutrino masses are too small to be observed in the direct search experiments, like the ^{3}H $\beta-$decay experiments.

The situation is very different if the three massive neutrinos are quasi-degenerate in mass, eq. (30). The massive neutrinos can be cosmologically relevant: one can have $m_i \cong$ few eV $\gg \sqrt{\Delta m_{atm}^2}$, i=1,2,3. Actually, the experimental upper limit (9) implies in this case $m_i < (2.5 - 3.0)$ eV. If the future ^{3}H $\beta-$decay experiments will observe an effect of nonzero neutrinos mass $\sim (1 - 3)$ eV, that would imply within the scheme (10) that the three massive neutrinos are quasi-degenerate. In the four-neutrino mixing schemes discussed in Section (4), neutrino masses in the range $\sim (1-3)$ eV are possible [37,44] in the scheme (A), but not in the scheme (B). A negative result of the ^{3}H experiments will not provide an information on the structure of the neutrino mass spectrum.

Additional information on the neutrino mass spectrum can be obtained in the future high sensitivity $(\beta\beta)_{0\nu}-$decay experiments, provided the massive neutrinos are Majorana particles [46]. For $m_i \ll$ MeV, i=1,2,3,4, which is the case of interest, we have for the neutrino mass parameter measured in the $(\beta\beta)_{0\nu}-$decay experiments (see, e.g., [9]):

$$< m_\nu > = \sum_{k=1} m_k \eta_k \ |U_{ek}|^2, \tag{31}$$

where $i\eta_k = \pm i$ is the CP-parity of the Majorana neutrino ν_k and we have written for simplicity the expression for $< m_\nu >$ in the case of CP-conservation. Because the massive Majorana neutrinos can have opposite CP-parities, there can be cancellation between the different terms in the sum in eq. (31).

We can use the values of the lepton mixing angles and of the masses m_k in the cases (28) - (30) required by the three possible solutions of the solar neutrino problem and the neutrino oscillation explanation of the atmospheric neutrino data together with the CHOOZ limit (7) to derive upper bounds on

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the value of $| < m_\nu > |$. One arrives in this way to the following conclusions [46]. If the spectrum is hierarchical (28), then $| < m_\nu > | \leq 0.008$ eV. In the case of two quasi-degenerate neutrinos (28) or (29), $| < m_\nu > |$ can be larger than 0.01 eV, but cannot exceed 0.10 eV. Finally, if (30) is realized, $| < m_\nu > |$ can have a value in the interval (0.10 - 1.0) eV. The latter is valid also in the case of 4-neutrino mixing in the scheme (A), while in scheme (B) one has $| < m_\nu > | < 0.01$ eV [37,44]. Thus, if the $(\beta\beta)_{0\nu}-$decay will be observed in the future experiments, the measurement of its rate can provide important information on the neutrino mass spectrum.

6 Conclusions

To summarize, the possible patterns of the lepton mixng are emerging from the analyses of the solar and atmospheric neutrino data. They are very different from the pattern of the quark mixing. Future ^{3}H $\beta-$decay and $(\beta\beta)_{0\nu}-$decay experiments can provide information on the neutrino mass spectrum. However, more data is needed to establish i) the true cause of the solar neutrino deficit (VO, or MSW transitions, or may be something else...), and ii) the type of dominant oscillations of the atmospheric ν_μ ($\nu_\mu \to \nu_\tau$ or $\nu_\mu \to \nu_s$). It is also very important to establish whether the sub-dominant $\nu_\mu \to \nu_e$ and $\nu_e \to \nu_{\mu,\tau}$ oscillations of the atmospheric neutrinos take place. These sub-dominant oscillations and the transitions of the solar neutrinos in Earth can be strongly (maximally) enhanced by a new type of mechanism which differs from the MSW one and takes place when the neutrinos cross the Earth core on the way to the detector. It would be quite remarkable to observe experimentally the indicated enhancement.

We believe at least some of the above questions will be answered by the future experiments: SNO, BOREXINO, ICARUS, HERON, etc. in the case of solar neutrinos, and by K2K, MINOS, KAMLAND, mini-BOONE, etc. The future in the field of physics of neutrino oscillations and massive neutrinos looks as exciting as the present.

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